A cross-technology benchmark for incremental graph queries

Georg Hinkel, Antonio Garcia-Dominguez, René Schöne, Artur Boronat, Massimo Tisi, Théo Le Calvar, Frederic Jouault, József Marton, Tamás Nyíri, János Benjamin Antal, Márton Elekes, Gábor Szárnyas

Presenter: Gábor Szárnyas (CWI Amsterdam)

Software and Systems Modeling 2022 | MODELS 2022 J1 track

TTC 2018 "Social media" case

Your solutions - Programme Calls - Aims and scope People History -



News

11th Transformation Tool Contest

A contest for users and developers of transformation tools.

Part of the Software Technologies: Applications and Foundations (STAF) federated conferences.

Hosted at the IRIT in Toulouse, France on Friday 29 June 2018.

"Social media" case

Scoring posts

Score = 10 × #comments + #likes



For each comment, find connected components of users who liked the comment



Score = Σ (component size)²



Score = Σ (component size)² = 1² + 1² = 2



Score = Σ (component size)² = 1² + 1² = 2



Score = Σ (component size)² = 3² = 9



Solutions

	Solution	Data	a model	Va	arian	ts	
	Active Operations Framework	EMF			1		
	ATL	EMF			2		
	Hawk	EMF			3		
	JastAdd	EMF	Мс	osi	so	luti	ions use the
	Xtend	EMF	Eclips	se	Mo	del	ing Framework
	YAMTL	EMF			3		
	NMF	NMF			2		
	Differential Dataflow	relati	onal		1	2	1 solutions in total
DRMSe	PostgreSQL	relati	onal		2		
DDIVIOS	Neo4j	graph	n		2		
	GraphBLAS	matri	X		2		

Non-incremental query formulation

Examples of the how the initial query evaluation is formulated in:

- NMF
- Neo4j
- PostgreSQL

Note: Implementations can be quite complex – this is a "programming contest"

Scoring posts

Score = 10 × #comments + #likes

Traversing the Submission tree



<u>NMF</u>



post.Descendants()

<u>Neo4j (Cypher)</u>



MATCH (p:Post)

OPTIONAL MATCH (p)<-[:REPLY_OF*]-(c:Comment)

PostgreSQL (SQL:1999)

6	Z

WITH RECURSIVE

comments_with_ancestors(id, ancestorid) AS (
 SELECT c.id, c.parentid AS ancestorid
 FROM comments c
 UNION
 SELECT cr.id, c.parentid AS ancestorid
 FROM comments_with_ancestors cr
 , comments c
 WHERE cr.ancestorid = c.id)

Score = Σ (component size)²

Finding connected components of Users



NMF: Tarjan's algorithm

```
let layering = Layering<IUser>.CreateLayers(
    comment.LikedBy,
    u => u.Friends.Intersect(comment.LikedBy))
let score = layering.Sum(l => Square(l.Count))
```

SOEO4j Neo4j: User-defined function

. . .

```
MATCH (c:Comment) WHERE (c)<-[:LIKES]-(:User)
CALL gds.wcc.stream({
  nodeQuery:
    "MATCH (c:Comment)<-[:LIKES]-(u:User)</pre>
     WHERE id(c) = " + id(c) + "
     RETURN id(u) AS id",
  relationshipQuery:
    "MATCH (u1:User)<-[:FRIENDS]->(u2:User)
     RETURN id(u1) AS source, id(u2) AS target",
  validateRelationships: false
})
YIELD componentId
```

Score = Σ (component size)²

Finding connected components of Users





. . .

PostgreSQL: Simplified SQL query

WITH RECURSIVE comment friends(commentid, user1id, user2id) AS (...), comment friends closed(commentid, head userid, tail userid) AS **SELECT** 1.commentid , l.userid AS head userid, l.userid AS tail userid FROM likes 1 UNTON SELECT cfc.commentid, cfc.head userid, f.user2id AS tail userid FROM comment friends closed cfc, comment friends f WHERE cfc.tail userid = f.user1id **AND** cfc.commentid = f.commentid), comment components AS (SELECT commentid, head userid AS userid , min(tail userid) AS componentid FROM comment friends closed GROUP BY commentid, head userid), comment_component_sizes AS (SELECT cc.commentid, cc.componentid, count(*) AS component size FROM comment components cc GROUP BY cc.commentid, cc.componentid SELECT c.id AS commentid , coalesce(sum(power(ccs.component size, 2)), 0) AS score FROM comments c **LEFT JOIN** comment component sizes ccs **ON** (ccs.commentid = c.id) **GROUP BY** c.id, c.ts

Incremental maintenance





Solution	Data model	Explicitly incremental	Implicitly incremental
Xtend	EMF	_	-
Hawk	EMF	+	-
PostgreSQL	relational	+	-
Neo4j	graph	+	-
GraphBLAS	matrix	+	-
Active Operations Framework	EMF	-	+
ATL	EMF	-	+
JastAdd	EMF	_	+
NMF	NMF	-	+
Differential Dataflow	relational	_	+
YAMTL	EMF	+	+

Score = Σ (component size)²

Finding connected components of Users



Incremental evaluation

The **granularity** of the incremental maintenance has a big effect on performance:

- New "likes" edge → recalculate only for the the affected single comment
- New "knows" edges → recalculate for each affected comments
- Reusing existing connected components?

Scoring comments: SQL incremental

1	INSERT INTO	comment_friends (status, mentid, user1id, user2id)
2	SELECT	'B' AS status
3	,	11.commentid, f.user1id,
		f.user2id
4	FROM	likes 11, likes 12
5	,	friends f
6	WHERE	<pre>l1.userid = f.user1id</pre>
7	AND	f.user2id = 12.userid
8	AND	<pre>l1.commentid = l2.commentid;</pre>

Listing 36 Initialization phase for the Incremental PostgreSQL solution for Q2, initializing the comment_friends relation.

1	INSERT INTO	comment_friends (status,
	con	mentid, user1id, user2id)
2	SELECT	'D' AS status
3	,	<pre>l1.commentid, f.user1id,</pre>
		f.user2id
4	FROM	likes_d 11, likes 12
5	,	friends f
6	WHERE	11.userid = f.user1id
7	AND	f.user2id = 12.userid
8	AND	11.commentid = 12.commentid
9	UNION ALI	5
10	SELECT	'D' AS status
11	,	<pre>l1.commentid, f.user1id,</pre>
		f.user2id
12	FROM	likes_b 11, likes 12
13	,	friends_d f
14	WHERE	l1.userid = f.user1id
15	AND	f.user2id = 12.userid
16	AND	11.commentid = 12.commentid
17	UNION ALI	2 · · · · · · · · · · · · · · · · · · ·
18	SELECT	'D' AS status
19	,	<pre>l1.commentid, f.userlid,</pre>
		f.user2id
20	FROM	likes_b 11, likes_d 12
21	,	friends_b f
22	WHERE	l1.userid = f.user1id
23	AND	f.user2id = 12.userid
24	AND	<pre>l1.commentid = l2.commentid;</pre>

1 WITH RECURSIVE comment friends closed init(commentid, head_userid, tail_userid) AS (3 -- transitive closure (reachability-only, no path is recorded) 4 -- of friendship-subgraphs defined by comment likes 5 -- start with the users that liked 6 a specific comment. 7 -- They are the nodes of the projected users graph for a comment 8 SELECT 1.commentid, 1.userid AS head userid, 1.userid AS tail userid 9 FROM likes 1 10 UNION 11 -- expand the closure with the edges of the projected graph, 12 -- which is stored in comment friends table 13 SELECT cfc.commentid. cfc.head userid, f.user2id as tail_userid 14 FROM comment friends closed init cfc 15 , comment_friends f 16 WHERE cfc.tail userid = f.userlid AND cfc.commentid = f.commentid 17 18) **19 INSERT INTO** comment friends closed(commentid, head_userid, tail_userid) 20 select commentid, head userid, tail userid 21 from comment_friends_closed_init w 22 left join comment_friends_closed q using (commentid, head_userid, tail userid) 23 where g.commentid IS NULL;

Listing 38 SOL initialization phase for the Incremental PostgreSOL solution for Q2: initializing the comment_friends relation's closure.

1	WITH comment_components AS (
2	SELECT commentid, head_userid AS
	userid
3	, min(tail_userid) AS
	componentid
4	FROM comment_friends_closed
5	GROUP BY commentid, head_userid
6)
7	, comment_component_sizes AS (
8	SELECT cc.commentid,
	cc.componentid, count(*) AS
	component_size
9	FROM comment_components cc
10	GROUP BY cc.commentid,
	cc.componentid
11)
12	consider all comments including
	those without likes
13	SELECT c.id AS commentid
14	, coalesce(sum(
	power(ccs.component_size,
	2)), 0) AS score
15	FROM comments c left join
	comment_component_sizes ccs
	on (ccs.commentid = c.id)
16	GROUP BY c.id, c.ts
17	ORDER BY sum (
	<pre>power(ccs.component_size, 2))</pre>
	DESC NULLS LAST
18	. c.ts DESC LIMIT 3:

Listing 40 SQL result retrieval phase for the Incremental PostgreSQL solution for O2.

1	WITH RECURSIVE note: though not the 1st query is
	the recursive one, the RECURSIVE keyword
	needs to be at the beginning
2	comment_friends_closed_stage0 AS (
3	in order to maintain the transitive closure in
4	we build on the transitive closure built so
	far and the new likes.
5	We need the new likes because users that liked a specific comment
6	are the nodes of the projected users graph for a comment
7	SELECT commentid, head userid, tail userid
8	FROM comment friends closed

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SELECT I. Commentid, I. userid AS head_userid,
<pre>l.userid AS tail_userid</pre>
FROM likes_d 1
)
, comment_friends_closed_stage1(commentid,
head_userid, tail_userid) AS (
the transitive closure computed so far
(reachability-only, no path is recorded)
is expanded by paths built from the new friendships
SELECT commentid, head_userid, tail_userid
FROM comment_friends_closed_stage0
UNION
SELECT cfc.commentid, cfc.head_userid, f.user2id
as tail_userid
FROM comment_friends_closed_stage1 cfc
, comment_friends_d f
WHERE cfc.tail_userid = f.userlid
AND cfc.commentid = f.commentid
, comment friends closed stage2 AS (
transitive closure having the new friendships is
then expanded using the
previous transitive closure stage
SELECT distinct of commential of head userid
r tail userid
FROM comment friends closed stagel of
inner join comment friends closed r on
(cfc tail userid -
(cic.tuii_userid =
ria compositid - a compositid)
trom forth and there is a well in
LEFT JOIN AND WHERE IS NOLL IS
the antijoin
used to eliminate edges already
present in the previous closur
this is to prevent unnecessary
CONFLICTS in the INSERT
statement below.
left join comment_friends_closed s0 on
(cfc.commentid = s0.commentid
AND cfc.head_userid =
s0.head_userid AND
cfc.tail_userid =
s0.tail_userid)
WHERE s0.commentid IS NULL
UNION
SELECT commentid, head_userid, tail_userid
FROM comment_friends_closed_stage1
INSERT INTO comment_friends_closed(commentid,
head_userid, tail_userid)
<pre>select commentid, head_userid, tail_userid</pre>
<pre>from comment_friends_closed_stage2 w</pre>
<pre>left join comment_friends_closed q using</pre>
(commentid, head_userid,
tail_userid)
where a commentid IS NULL

45 ON CONFLICT DO NOTHING:

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41 42

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Listing 39 SQL maintenance phase for the Incremental PostgreSQL solution for Q2: updating the comment_friends relation's closure.

Listing 37 SQL maintenance phase for the Incremental PostgreSQL solution for Q2: updating the comment_friends relation.

Results and findings



Scoring posts









Findings

- 1. Implicitly incremental tools are superior
- 2. Lacklustre performance from databases
- 3. Parallelization is not supported by EMF tools and databases
- 4. User-defined functions are important
- 5. Fair benchmarking and reproducibility are challenging

The Linked Data Benchmark Council

The Linked Data Benchmark Council (LDBC) is a non-profit organization founded in 2012 with members from academia and industry. Its goals are:

- 1. Defining graph processing benchmarks
- 2. Facilitating fair competition
- 3. Accelerating the adoption of ISO GQL and SQL/PGQ

LDBC members

20 companies and organizations, including:

LDBC benchmarks

Social Network Benchmark:

• The data of the TTC 2018 "Social Media" case is a subset of the SNB.

The SNB has new and updated workloads:

- analytical: Business Intelligence workload v1
- transactional: Interactive workload v2

Rigorous auditing process that takes system costs (license and ownership) into account.

Conclusion

Conclusion

The TTC 2018 "Social Media" case:

- A cross-technology benchmark
- for incremental graph queries

Findings:

- Two simple graph queries can be challenging to formulate even non-incrementally
- DBMSs have performance issues for graph queries
- Explicit incremental evaluation is difficult
- Implicit incremental tools are superior
- → "retrofitted" incrementality has limited benefits

Finding: Lack of parallelization

Parallelization is paramount today: even laptop CPUs have 8–16 cores.

The initial evaluation is trivially parallelizable for both queries.

Observation:

- Only NMF, Differential Dataflow, and GraphBLAS support parallelization.
- EMF tools and databases (Neo4j, PostgreSQL) lack parallelization.

Finding: Importance of user-defined functions

Some computations are difficult to express in a declarative language, e.g. the connected components algorithm

User-defined functions (UDFs) can be used to express these computations

- Common among EMF tools Java/Xtend code operating on the EMF model
- Database systems like Spark/Databricks and Snowflake support Java UDFs

Incremental maintenance of UDFs is difficult

Finding: Fair benchmarking is difficult

We are comparing very different systems:

- EMF tools
- Neo4j graph DBMS
- PostgreSQL relational DBMS
- GraphBLAS concurrent sparse linear algebra library written in C
- Differential Dataflow dataflow library written in Rust

Reproducibility is also difficult:

- dockerized execution
- extensive CI tests
- benchmarking in standard cloud VMs

Limitations of the benchmark

Limitations

No delete operations

• Adding them to the data generator is difficult (GRADES-NDA'20 paper)

No (unweighted) shortest path queries

• Another important graph kernel, also challenging with deletes

See examples in the next slides.

Connected components with delete operations

Scoring comments

Score = Σ (component size)² = 3² = 9

Connected components with delete operations

Scoring comments

Score = Σ (component size)² = $1^2 + 1^2 = 2$

Shortest path: [u1, u5, u4]

Shortest path: ?

Shortest path: [u1, u2, u3, u4]

Shortest path: [u1, u2, u3, u4] [u1, u2, u5, u4]

Ideas for incremental evaluation // Future work

Ideas for incremental evaluation

The ideas in the following slides could work **if all inserts are added one-by-one** and there are not too many inserts. (There aren't, see the table with the <u>model sizes</u>.)

IIRC none of the solutions in the paper used this: they all went for a bulk insertion followed by a single recomputation step.

With a client-server setup, doing operations in bulk likely makes sense. Performing an individual maintenance operation per insert is likely expensive. With a read-oriented system (e.g. column store), it makes sense to perform the inserts in bulk.

Still, it would be interesting to give this a go with an embeddable database (e.g. DuckDB, Neo4j) or a system which provides an option to write stored procedure (like Oracle's PL/SQL).

Scoring posts

Score = 10 × #comments + #likes

Traversing the Submission tree

Trick: For each Comment, store its root Post. When inserting a new child Comment, it should get its parent's root Post.

This works because the subgraph is a tree and there are no cut-and-link operations.

Trick: Upon adding a new "likes" or a new "friends" edge, connected components can only be merged together.

This works because there are no delete operations.

Scoring comments

Score = Σ (component size)²

Finding connected components of Users

Trick: Upon adding a new "likes" or a new "friends" edge, connected components can only be merged together.

This works because there are no delete operations.

Scoring comments

Score = Σ (component size)²

Finding connected components of Users

Potential extensions to the slide deck

- concrete result slides
- details on SQL/PGQ
 WONTFIX
- model sizes
- incremental query formulation
- interesting findings
- more info on concrete tools
 - mention of DD & videos
 - mention of GraphBLAS
 - Hawk, NMF, YAMTL, etc.
- anything on DuckDB/DuckPGQ as a potential tool for Q1
- complaining that most MDE tools are single-threaded
- incremental tricks explained...

Incremental query formulation

Incremental view maintenance

Categories:

- Non-incremental: query is recomputed each time
- Implicitly implemental: the maintenance is done automatically by the system
- **Explicitly incremental:** the query developer manually incrementalizes the query (poor man's view maintenance, can be retrofitted to existing systems)

Studied in depth in database research

...but most research focused on equijoins

...maybe anti- and outer joins

Transitive reachability (tree queries, connected components, etc.) are less studied.

All tools

- (1) AOF
- (2) ATL (3) + Incremental
- (4) Differential Dataflow
- (5) GraphBLAS (6) + Incremental
- (7) Hawk (8) + IU (9) + IUQ
- (10) JastAdd
- (11) Neo4j (12) + Incremental
- (13) NMF (14) + Incremental
- (15) PostgreSQL (16) + Incremental
- (17) Xtend
- (18) YAMTL (19) + II (20) + EI

```
Most tools use the EMF data model
(1) Active Operations Framework (AOF)
(2) ATL (3) + Incremental
(7) Hawk (8) + IU (9) + IUQ
(10) JastAdd
(17) Xtend
(18) YAMTL (19) + II (20) + EI
```

```
NMF:
(13) NMF (14) + Incremental
```

Relational: (4) Differential Dataflow (15) PostgreSQL (16) + Incremental

Property graph: (11) Neo4j (12) + Incremental

Matrix: (5) GraphBLAS (6) + Incremental

Draft

a very small benchmark suite, just two queries and a few transformations

already highlights numerous usability and performance characteristics of systems !!

e.g. why don't MDE tools use relational DBMSs

discuss the two queries briefly

present a few solutions (e.g. Postgres/SQL; Neo4j/Cypher; MDE tools; differential dataflow, refer to Frank McSherry's video)

FMS videos

Live coding differential dataflow:

- https://www.youtube.com/watch?v=W6TKxS_pWr0
- https://www.youtube.com/watch?v=83rG471bmw8
- https://www.youtube.com/watch?v=uZ23MnpujNA

<u>Schema</u>

Differential dataflow: CC computation

```
likes
                        // node
                                   label
                                           comment
    .filter(| | false)
    .map(|(user, comm)| ((user.clone(), comm), user))
    .iterate(|labels| {
        let knows = knows.enter(&labels.scope());
        let likes = likes.enter(&labels.scope());
        labels
            .map(|((node, comment), label)| (node, (label, comment)))
            .join map(&knows, | node, (label, comment), dest| ((dest.clone(), comment.clone()), label.clone()))
            .concat(&likes.map(|(user, comm)| ((user.clone(), comm), user)))
            .reduce(|_key, input, output| {
                // only produce output, if `input` contains `_key.0`
                if input.iter().any(|(label,_wgt)| *label == &_key.0) {
                    output.push((input[0].0.clone(), 1));
                }
            })
    });
```

Solution	Incremental conn. components	Algorithm	Sorting
AOF	8	Breadth-first traversal	Incremental
ATL	0	Depth-first traversal	Full
ATL incremental	\otimes	Breadth-first traversal	Incremental
Differential Dataflow	\otimes	Fixed-point label propagation	Full
GraphBLAS	0	FastSV	Offline top- <i>x</i>
GraphBLAS incremental	0	FastSV on overestimation + merge	Online top- <i>x</i>
Hawk	0	Tarjan	Full
Hawk (IU)	0	Tarjan	Full
Hawk (IUQ)	0	Tarjan	Online top- x
JastAdd	0	Depth-first/Kosaraju	Offline top- <i>x</i>
Neo4j	0	Union-find variant	Offline top- <i>x</i>
Neo4j incremental	\otimes	Breadth-first traversal	Incremental
NMF reference	0	Tarjan	Full
NMF incremental	\otimes	Edge changes	Incremental
PostgreSQL	0	Breadth-first traversal	Online top- <i>x</i>
PostgreSQL incremental	0	Overestimation + breadth-first traversal	Online top- x
Xtend	0	Tarjan	Online top- <i>x</i>
YAMTL-B	0	Weighted quick-union-find with path compression	Online top- <i>x</i>
YAMTL-II	\otimes	Weighted quick-union-find with path compression	Online top- <i>x</i>
YAMTL-EI	\otimes	Weighted quick-union-find with path compression	Online top- <i>x</i>

 Table 2
 Classification of approaches for Q2, sorted by tool and incrementality

Notation— \otimes yes; \otimes to some extent; \bigcirc no. Overestimation means that all connected components are re-computed that might be affected by the changes

Solution	Incrementality	Sorting
AOF	\otimes	Incremental
ATL	0	Full
ATL incremental	\otimes	Incremental
Differential Dataflow	\otimes	Full
GraphBLAS	0	Offline top- <i>x</i>
GraphBLAS incremental	\otimes	Offline top- x
Hawk	0	Full
Hawk (IU)	0	Full
Hawk (IUQ)	0	Online top-x
JastAdd	0	Offline top- <i>x</i>
Neo4j	0	Offline top- <i>x</i>
Neo4j incremental	\otimes	Incremental
NMF reference	0	Full
NMF incremental	\otimes	Incremental
PostgreSQL	0	Online top-x
PostgreSQL incremental	\otimes	Online top- <i>x</i>
Xtend	0	Online top-x
YAMTL-B	0	Online top-x
YAMTL-II	0	Online top- <i>x</i>
YAMTL-EI	\otimes	Online top- <i>x</i>

Table 1 Classification of approaches for Q1, ordered by solution nameand support for incrementality

Notation: \otimes yes; \otimes to some extent; \bigcirc no

Tool	Version	Data model	Engine	Solution	Decl.	Batch	Implicit	Explicit	DB	MV	Parallel
AOF	v201806	EMF	Java 8	Xtend	0	0	\otimes	0	0	0	0
ATL	3.8.0	EMF	Java 8	ATL	\otimes	\otimes	0	0	0	0	0
ATL Incremental	v201904	EMF	Xtend	ATL/AOF	\otimes	0	\otimes	0	0	0	0
Differential Dataflow	0.11.0	relations	Rust	Rust	0	0	\otimes	0	0	0	\otimes
GraphBLAS	4.0.3	matrices	С	C++	0	\otimes	0	\otimes	0	0	\otimes
Hawk	2.1.0	EMF	Java 8	EOL	\oslash	\otimes	Ø	\otimes	\otimes	Ø	0
JastAdd	2.3.5	EMF	Java 11	Java 11	0	\otimes	\otimes	0	0	0	0
Neo4j	4.2.4	property graph	Java 11	Java 11	\otimes	\otimes	0	\otimes	\otimes	\otimes	0
NMF	2.0.169	NMF	C#	C#	\otimes	\otimes	\otimes	0	0	0	\otimes
PostgreSQL	12.4	relations	С	Java 11/SQL	\otimes	\otimes	0	\otimes	\otimes	\otimes	0
Xtend	2.20.0	EMF	Java 8	Xtend	\otimes	\otimes	0	0	0	0	⊛
YAMTL	0.1.5	EMF	Java 11	Xtend	\oslash	\otimes	\otimes	\otimes	0	0	0

 Table 4
 Comparison of the tools used in the paper

Data Model: the data model exposed to the user. *Engine:* programming language used to implement the engine (model transformation engine, database query engine, etc.), *Solution:* programming language and query language (if applicable) used to implement the solution, *Decl.:* the solution specified the queries using a declarative query language, *Batch:* only batch mode is supported, *Implicit:* implicit incrementalization is supported, *Explicit:* a solution with explicit incrementalization was implemented, *DB:* database-backed, i.e. the tool persists the model on disk after each transaction, *MV:* materialized views, *Parallel:* parallelization is supported. *Notation*— \otimes yes; \oslash to some extent; \bigcirc no; \circledast yes, using Java 8 streams

Model sizes for each scale factor

Type\scale factor	1	2	4	8	16	32	64	128	256	512	1024
Comments	640	1064	2315	5056	9220	18,872	39,212	76,735	148,470	273,418	540,905
Posts	554	889	1845	2270	5518	10,929	18,083	37,228	74,668	167,299	314,510
Users	80	118	190	204	394	595	781	1158	1678	2606	3699
Total number of nodes	1274	2071	4350	7530	15,132	30,396	58,076	115,121	224,816	443,323	859,114
friends	53	102	262	298	904	1827	2752	5695	11,118	24,387	45,386
replyTo	640	1064	2315	5056	9220	18,872	39,212	76,735	148,470	273,418	540,905
likes	6	24	66	129	572	1598	4770	13,374	36,815	102,276	268,432
submitter	1194	1953	4160	7326	14,738	29,801	57,295	113,963	223,138	440,717	855,415
Total number of edges	2533	4207	9118	17,865	34,654	70,970	143,241	286,502	568,011	1,114,216	2,251,043
Total number of changes	67	120	132	104	110	117	68	86	45	112	74

 Table 7
 Model sizes for each scale factor: number of nodes and edges, number of changes